Probability Distributions

Discrete Distributions

Binomial

$$P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y} \text{ for } y = 0, 1, ..., n$$
$$E(Y) = np \quad \text{Var}(Y) = np(1-p)$$

Geometric

$$P(Y=y) = p(1-p)^y \text{ for } y = 0, 1, 2, ...$$
$$E(Y) = \frac{1-p}{p} \quad \text{Var}(Y) = \frac{1-p}{p^2}$$

Poisson

$$P(Y=y) = \frac{e^{-\lambda}\lambda^y}{y!} \text{ for } y = 0, 1, 2, ...$$
$$E(Y) = \lambda \quad \text{Var}(Y) = \lambda$$

Hypergeometric

$$P(Y=y) = \frac{\binom{M}{y}\binom{N}{n-y}}{\binom{M+N}{n}} \text{ for } y = 0, ..., \min(M, n)$$
$$E(Y) = \frac{Mn}{M+N} \quad \text{Var}(Y) = n\frac{MN}{M+N}\frac{M+N-n}{M+N-1}$$

Negative Binomial

$$P(Y = y) = \binom{y+r-1}{y} p^r (1-p)^y \text{ for } y = 0, 1, 2, ...$$
$$E(Y) = \frac{r(1-p)}{p} \quad \text{Var}(Y) = \frac{r(1-p)}{p^2}$$

Continuous Distributions

Exponential

$$f(y) = \frac{1}{\theta} e^{-\frac{1}{\theta}} \text{ for } y > 0$$
$$F(Y) = 1 - e^{-\frac{y}{\theta}} \text{ for } y > 0$$
$$E(Y) = \theta \quad \text{Var}(Y) = \theta^2$$

Gamma

$$f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y} \text{ for } y \geq 0$$
$$E(Y) = \frac{\alpha}{\beta} \quad \text{Var}(Y) = \frac{\alpha}{\beta^2}$$

Beta

$$f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1} (1-y)^{\beta-1} \text{ for } y \in (0, 1)$$
$$E(Y) = \frac{\alpha}{\alpha+\beta} \quad \text{Var}(Y) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$$

Normal

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$
$$E(Y) = \mu \quad \text{Var}(Y) = \sigma^2$$
